Indifference Bands for Route Switching

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Abstract

The replacement I-35W bridge in Minneapolis saw less traffic than the original bridge though it provided substantial travel time saving for many travelers. This observation cannot be explained by the classical route choice assumption that travelers always take the shortest path. Accordingly, a boundedly rational route switching model is proposed assuming that travelers will not switch to the new bridge unless travel time saving goes beyond a threshold or "indifference band". To validate the boundedly rational route switching assumption, route choices of 78 subjects from a GPS travel behavior study were analyzed before and after the addition of the new I-35W bridge. Indifference bands are estimated for both commuters who were previously bridge users and those who never had the experience of using the old bridge. This study offers the first empirical estimation of bounded rationality parameters from GPS data and provides guidelines for traffic assignment.

Keywords:
Bounded rationality, Indifference band, Empirical estimation, GPS study, Route Choice

1. Introduction

The I-35W Mississippi River bridge plays a critical role in transporting commuters to downtown Minneapolis and the University of Minnesota. Its collapse in 2007 forced 140,000 daily users (Guo and Liu 2011; Zhu et al. 2010) to switch to other parallel bridges or to cancel their trips. Accordingly, the I-94 Bridge was restriped with one more lane to relieve traffic pressure across the river. A year later, a replacement I-35W bridge was rebuilt over the same location and the extra lanes on I-94 were closed. The addition of the bridge offered commuters another option to cross the river. Surprisingly, only 100,000 daily trips on average were observed on the new bridge (He and Liu 2012; Zhu 2011). According to Zhu (2011), the total travel demand in the Minneapolis-St. Paul metropolitan area dropped slightly in 2008 due to the economic crisis, but not enough to explain this fall-off. In contrast, daily trips on the I-94 Bridge returned to the original level before the I-35W bridge collapsed. Therefore, we assume that variation in travel demands is not the main reason for the significant traffic reduction on the replacement bridge.

To further understand the aforementioned phenomenon, two major GPS-based studies, including 143 commuters whose route choices might be affected by the addition of the new link were conducted (Carrion and Levinson 2012; Zhu 2011; Zhu and Levinson 2012). These commuters' trips were tracked by GPS two to three weeks before and eight to ten weeks after the reopening of the new I-35W bridge. Each commuter's day-to-day commuting routes and associated travel time could be drawn from GPS data. By comparing route choices before and after the new bridge was rebuilt, it is posited that commuters' "stickiness of driving habit" Zhu (2011) prevented them from taking the new bridge and thus resulted in a traffic flow drop on the new bridge. There may also be perception errors at work. Moreover, Zhu (2011) further calculated travel time differences between the routes actually taken and the shortest time routes from GPS data. Fewer than

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40% of commuters took the shortest paths, though 90% of subjects took routes which were within 5 minutes of the shortest paths and almost no commuter chose a route 20 minutes longer than the shortest path.

Utilizing the same GPS dataset, Carrion and Levinson (2012) modeled the time a commuter consistently left his current bridge choice for other alternative bridges and found that commuters chose routes based on a specific threshold and might abandon a route if its travel time exceeded the margin. This threshold depended on the social-demographics of subjects and varied day-to-day.

Previous studies focused on estimating indifference bands from laboratory experiment data. For example, Mahmassani and Chang (1987) estimated indifference bands by utilizing laboratory experiment data. By comparing commuter departure time and route choice switch behavior in laboratory experiments with field surveys in Dallas and Austin, Texas, Mahmassani and Jou (2000) showed that boundedly rational route choice modeling observed from experiments provided a valid description of actual commuter daily behavior. However, whether laboratory experimental experiences can represent actual commuter daily behavior still remains unclear.

Other studies incorporated given values of the thresholds into their route choice models, such as 20% of the mean travel time or 0.5 times the mean travel time of certain previous trips (Carrion and Levinson 2012; Yanmaz-Tuzel and Ozbay 2009; Zhu and Levinson 2012). These thresholds were obtained from experience or assumptions and served as inputs of specific route choice behavior models. Therefore the adoption of a given value may not be valid.

People do not choose routes irrationally. Yet, the empirical evidence argues that travelers do not always select the shortest travel time paths, but the chosen routes are within some threshold from the shortest ones. This paper examines the phenomenon of boundedly rational route choice behavior with GPS travel data collected in Minneapolis in 2008 (Carrion and Levinson 2012; Zhu 2011; Zhu and Levinson 2012). The disruption and the rebuilding of the I-35W bridge in Minneapolis provides us a rare opportunity to use GPS travel survey data to study route choices in response to the change in road network's topology. This study offers the first empirical estimation of bounded rationality parameters from GPS data.

The rest of the paper is organized as follows: The next Section focuses on the theoretical background of route choice. In Section 3, we discuss the details of the GPS data and present two categories of commuters of interest. Trip distribution among bridges over the Mississippi River is also presented. In Section 4, travel time saving by taking the new I-35W bridge is calculated based on a speed map pooled from GPS commuting trips. In Section 5, the boundedly rational route choice model is presented and we will show that subjects who used the old I-35W bridge display different behavioral patterns compared to those who never used the old bridge. Accordingly, in Section 6, indifference bands for old-users and non-users are estimated separately using GPS travel survey data. Conclusions and future research directions are discussed in Section 7.

2. Theoretical Background

2.1. Boundedly rational route choice behavior

In practice, most transportation planning software packages employ route choice algorithms based on Wardrop's first principle Wardrop (1952) that people take the shortest path(s) when traffic assignment is performed. In the academic literature, route choice is often considered within the framework of random utility maximization (RUM), each driver is assumed to take the route with the maximum utility among a set of finite paths. Each path is attached with several attributes, including travel time, distance, overlap with other paths, reliability, the number of traffic lights and turns, weather, scenery and so on. Provided perception errors are Gumbel or normal distribution, stochastic user equilibrium model can be expressed in the form of a multinomial logit or probit model.

One critique of both Wardrop's first principle and RUM is the assumption that people are "utility maximizers" (where utility is either travel time (Wardrop) or a bundle of factors including travel time (RUM)). Alternative hypotheses that are empirically based posit that people actually make decisions by strategies (Simon 1982), heuristics (Conlisk 1996), elimination by aspects (Tversky 1972), norms (Conlisk 1996) and/or rules (Nakayama et al. 2001; Lotan 1997), rather than solving an optimization.

Therefore people do not always choose the alternative with the maximum utility. Evidence from revealed route choice behavior finds after evaluating habitual routes, only 59% of respondents from Cambridge, Massachusetts (Bekhor et al. 2006), 30% from Boston (Ramming 2001), and 86.8% from Turin, Italy (Prato and Bekhor 2006) chose paths with the shortest distance or shortest travel time. According to GPS studies, 90% of subjects in the Minneapolis-St. Paul region took paths one-fifth longer than average commute time (Zhu 2011) and a high percentage of commuting routes were found to differ considerably from the shortest paths in Nagoya, Japan (Morikawa et al. 2005) and Lexington, Kentucky (Jan et al. 2000). All findings above revealed that people do not usually take the shortest paths and the utilized paths generally have higher costs than shortest ones.

To relax the unrealistic assumption that only the shortest paths are used, several route choice behavior models were proposed (Zhang 2011). This paper examines one alternative of the existing route choice behavior theories, i.e., bounded rationality. It says that people do not always select the shortest paths, but the chosen routes are within some threshold from the shortest ones.

As opposed to 'rationality as optimization', Herbert Simon, in 1957, proposed that people are boundedly rational in their decision-making processes (Simon 1957). This is either because people lack accurate information, or they are incapable of obtaining an optimized decision due to complexity of the situation. They tend to seek a satisfactory choice solution instead. Since then, bounded rationality has been studied extensively in economics and psychology. Bounded rationality in decision-making may also result from habit and inertia. People "place higher value on an opportunity if it is associated with the status quo" (Samuelson and Zeckhauser 1988), because it can provide significant energy saving to cognitive thinking. A large amount of empirical evidence finds that habit plays a significant role in behavior in stable situations (Bamberg and Schmidt 2003).

In travel behavior study, a series of experiments was conducted in the 1990s to empirically validate bounded rationality (Hu and Mahmassani 1997; Jayakrishnan et al. 1994; Mahmassani and Chang 1987; Mahmassani and Jayakrishnan 1991; Mahmassani and Liu 1999; Srinivasan and Mahmassani 1999). These experiments were run on an interactive simulator – DYNASMART, incorporating pre-trip departure time, route choices and en-route path switching decisions. Subjects, as travelers, picked departure time pre-trip based on previous days' travel experiences and chose paths en-route at each node based on available information. The experimental results showed that, in the repeated learning process, as a result of habit, commuters would not adjust their departure time unless the difference between preferred arrival time and actual arrival time exceeded a bound (Chen and Mahmassani 2004; Hu and Mahmassani 1997; Jayakrishnan et al. 1994; Mahmassani and Chang 1987; Mahmassani and Jayakrishnan 1991; Mahmassani and Liu 1999; Srinivasan and Mahmassani 1999). This bound for lateness and earliness differed and people were usually more sensitive to lateness.

Bounded rationality was also found in mode choices. Cantillo et al. (2006, 2007) indicated that there existed a threshold when the impact of the transportation planning policy change was evaluated on choice behavior. Travelers would not switch to a new mode unless its utility was greater than that of the current mode plus a threshold, which was a function of the difference between two experienced mode utilities. Then a discrete choice model with thresholds was applied to simulated SP/RP datasets to estimate and predict mode choice. The prediction results showed that a model without considering inertia overestimated benefits of transport investments substantially.

2.2. Threshold stimulus-response models

The existence of a threshold in choice behavior has also long been explored in other fields and the model describing this behavior is called "threshold stimulus-response" model. The stimulus-response model, popular in biology, psychology and economics, provides an efficient method of quantifying behavioral response by varying stimulus of specific intensities.

Biologists are interested in dose-response, which explores the impact of toxic levels on an organ or a tissue. In psychology and economics, stimulus-response studies focus on the change in decision-makers' preferences or choices in response to the change in utilities of alternatives. The occurrence of a response depended on the intensity of a stimulus and there existed a threshold under which no response was manifest (Cox 1987; Krishnan 1977). This threshold was named "just noticeable difference" by Weber or "minimum perceivable difference" by Krishnan (1977).

When the response is qualitative, such as dose quantity, Weber's law revealed that the response intensity is proportional to the logarithm of the stimulus. If the response is discrete, such as choices or preferences, several biological experiments (Clark 1933; Hemmingsen 1933) verified that no response occurred unless the logarithm of stimulus exceeded some threshold. Several models, such as the threshold dose-response models (Cox 1987), minimum perceivable difference model (Krishnan 1977), and biological probit or logit model (Krishnan 1977), were proposed to estimate the threshold. These models will be briefly discussed subsequently.

- Dose-response models (Cox 1987) assumed that the probability of response is zero if the amount of dosage is below a threshold parameter and follows logit or probit model if it is more than the threshold. Cox (1987) showed that the threshold model fit data better than traditional logit or probit model.
- The minimum perceivable difference model (Krishnan 1977) assumed indifference between two alternatives if the difference of their values falls within a threshold. Therefore, a third relation (i.e., indifference) other than 'greater than' or 'less than' was introduced. The threshold parameter was estimated via maximum likelihood method.
- In the biological probit or logit model (Krishnan 1977), the threshold is assumed to be a random variable with normal or logistic distribution and therefore a probit or a logit regression can be used to estimate distribution related parameters. The biological probit model was shown to predict responses more accurately than the logit model.

This paper employs threshold stimulus-response framework to model route choice behavior and estimate associated thresholds.

3. Route choice observations

GPS studies (Zhu 2011) provide the following data for 143 subjects:

- Each subject's home and work locations;
- Each subject's day-to-day commuting routes;
- Each subject's day-to-day travel time.

Most subjects use more than one path and switch routes from time to time. Therefore we define a "commonly chosen route" as the route a commuter uses most frequently during the study period. The commonly chosen route from the beginning of the study period up to September 18, 2008 (the day when the replacement I-35W bridge was opened) is the "before-route", and the one from September 18, 2008 until the end of the study is the "after-route".

Remark. 1. The same routes are defined as the ones which overlap at least 95% in length and start within a 600 m (approximately 4 city blocks) radius from home and end in a 600 m radius from the work location.

2. For each commuter, his or her experienced travel time on a path varies from day to day due to uncertainty in traffic conditions. Therefore "average travel time" on a before-route or on an after-route for a commuter is computed as the mean of day-to-day GPS measured travel time on that route when he or she uses it.

3.1. Subject classification

A "crosser" is the commuter whose home and work locations are on different sides of the river. Specifically, a crosser's commuting route options may be enlarged by the addition of the new bridge. Otherwise the subject is a "non-crosser". In general a non-crosser's route options are not enlarged by the existence of the new bridge. (Note: some individuals, nominally non-crossers, but in fact "double-crossers", may have crossed the river twice on certain routes from home to work, those individuals have been removed from the analysis, as the sample size was too small. ("Triple-crossers" etc. were not observed.))

The addition of the new bridge may or may not save crossers' average travel time. Denote $C_b^{(n)}, C_a^{(n)}$ as average travel times experienced by commuter n before and after the reopening of the new bridge. When the average travel time on the after-route minus that on the before-route is less than zero, i.e., $C_a^{(n)} - C_b^{(n)} < 0$, we say that the addition of the new bridge saves commuter n's travel time, or else it does not.

Table (1) shows the number of crossers and non-crossers. The "Change" column refers to those commuters who change their routes after the addition of the bridge, i.e., their before-routes and after-routes are different. Or else they belong to the "No change" column. The "Save time" row refers to those whose average travel time on after-routes is shorter than that on before-routes.

Table 1: Statistics of crossers and non-crossers

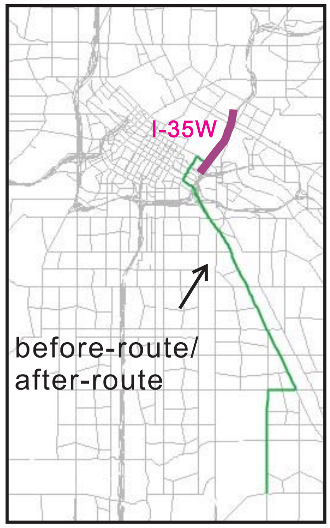
Type		Change	No change	Total (excluding missing data)	Subjects with incomplete observations	Total (including missing data)
		C1	C2	C3=C1+C2	C4	C5=C3+C4
Non-crosser	Save time	5	16	21	0	30
	Save no time	0	9	9		
Crosser	Save time	47	31	78	19	113
	Save no time	2	14	16		
Total		54	70	124	19	143

Remark. There are 19 crossers whose route choice observations before the addition of the bridge were missing, therefore they are classified as "Subjects with incomplete observations" in column $C4$.

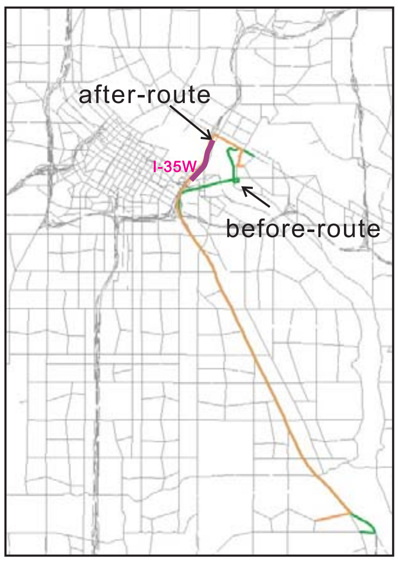
When the addition of the new bridge saves crossers' commuting time, a crosser chooses either to change to the new bridge or not. Thus we further divide those crossers into the following two categories:

- A "Switcher" is a crosser who switches to the new I-35W bridge as his or her after-route given his or her travel time can be shortened by the new bridge;
- A "Stayer" is a crosser whose travel time can be improved by the new bridge but stays on his or her before-route, i.e., the before-route and the after-route are the same.

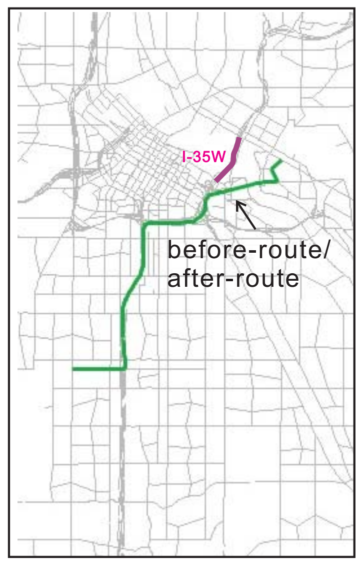
Figure (1) illustrates examples of a non-crosser, a switcher, a stayer and a crosser with no time saving. The I-35W bridge is indicted by the purple line. In Figure (1a), the non-crosser's before-route and after-route are the same because he does not need to cross the river and thus his route is not influenced by the new bridge. In Figure (1b-1c), the switcher uses two different routes before the bridge was rebuilt and after, while the stayer uses the same route which is not via the I-35W bridge. In Figure (1d), taking the new bridge cannot improve the crosser's travel time, so he stays on the same route.



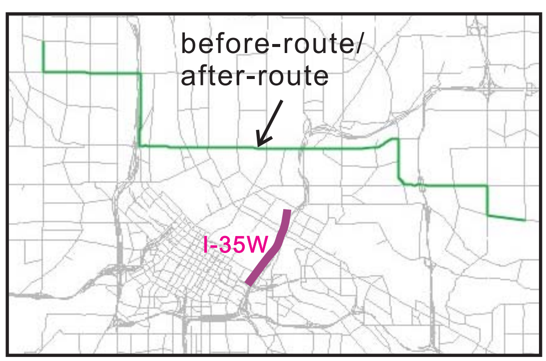
(a) Non-crosser



(b) Switcher



(c) Stayer



(d) Crosser without time saving

Figure 1: Examples of non-crosser, switcher, stayer and crosser with no time saving

In this study, only 47 switchers and 31 stayers are considered. So there are 78 subjects of interest.

3.2. Bridge usage analysis for switchers and stayers

Figure (2) shows eleven bridges, indicated by red with names next to them, used by subjects across the Mississippi River before and after the new bridge's reopening. The background is the TLG network (generated and maintained by Metropolitan Council and The Lawrence Group) which encompasses the entire seven-county Minneapolis-St. Paul Metropolitan Area.

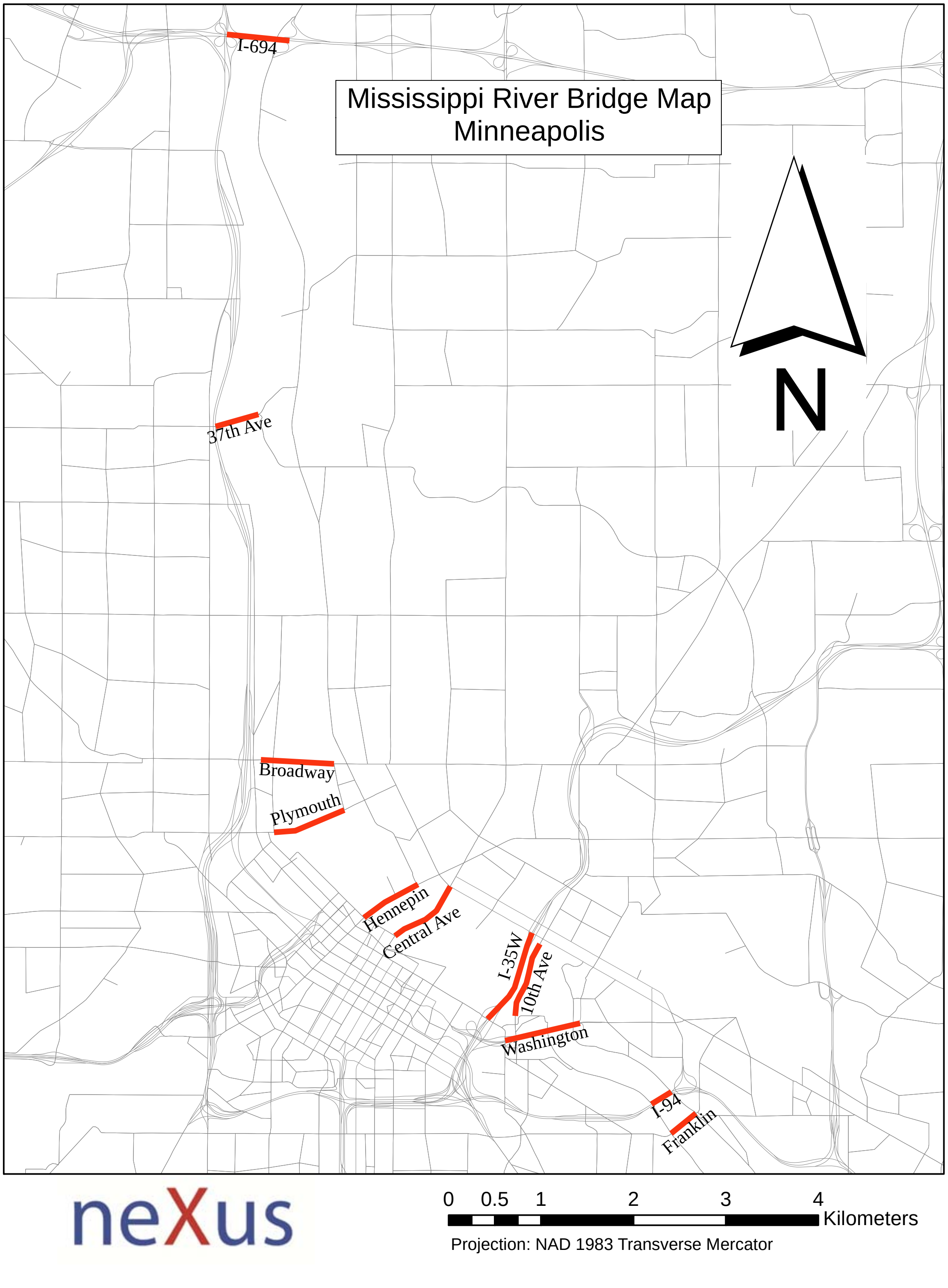


Figure 2: Bridges over the Mississippi River

Seventy-eight subjects made 2,167 morning commuting trips during the study period. Most subjects use more than one bridge to cross the river. The bridge used by the most trips is the "most frequently used bridge".

Figure (3) illustrates trip distribution among bridges. Before the new bridge was built, Washington, 10th Avenue and I-94 are the three most used bridges in the sample. After its reopening, the I-35W bridge is the main bridge carrying 64% of observed cross-river trips in the sample.

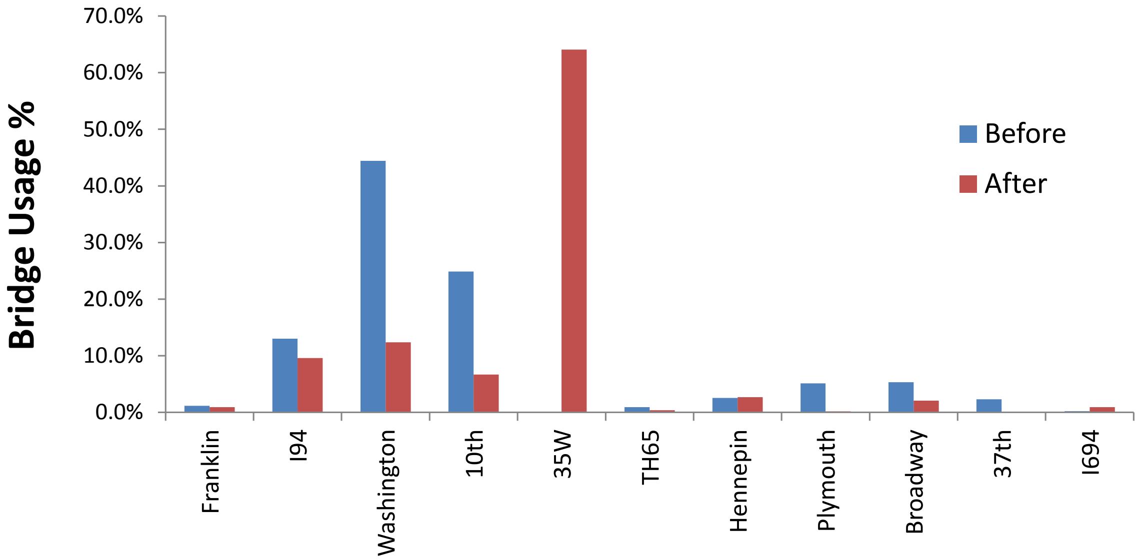


Figure 3: Cross-river trip distribution among bridges for study subjects

For 47 switchers, we further calculate the percentage of switchers who changed from their frequently used bridges to the I-35W bridge. Switchers originally on the 10th Avenue Bridge provide the highest portion of switching to the I-35W bridge and those originally on the Washington Ave Bridge follows. This is reasonable because the 10th Avenue Bridge and the Washington Ave Bridge are the nearest to the I-35W bridge.

We also compute the duration it took subjects to settle on a bridge after the reopening, i.e., from September 18, 2008 until the day when the subject have been taking the *most frequently used bridge* for at least two times consecutively (Carrion and Levinson 2012). Zero day for switchers means they immediately use the new bridge on September 18, 2008; while zero day for stayers means they stick to their before-routes regardless of the addition of the bridge. On average, it took 3.0 days for 78 subjects to stabilize their bridge choices. The number of days for switchers to stabilize is slightly longer than that for stayers and it has larger variation. Interested readers can refer to Carrion and Levinson (2012) which employed duration analysis to analyze how many days it took commuters to use the current bridge choice.

Table 2: Descriptive statistics of duration of bridge stabilization

	Mean	Std	Median	Min	Max
Switcher	3.2	6.0	0	0	27
Stayer	2.7	5.7	0	0	20
Total	3.0	5.8	0	0	27

4. Travel time saving calculation

In this study, we assume that travel time saving is the only stimulus for commuters to switch to the new bridge disregard of travel time reliability and other factors.

To obtain the travel time savings brought by taking the new I-35W bridge, we need to identify routes via the new bridge. For switchers, the after-route is the route via the new bridge. Stayers, on the other hand, never use the new bridge and therefore a route via I-35W bridge for each stayer should be first identified.

The speed map was pooled from $6,059$ commuting trips out of $25,157$ total trips. Only links with more than 5 observations before and after the new bridge's reopening were included. The average link speed was estimated from GPS data of all probe vehicles passing this link during the experiment period. This map covers a high portion of the freeway system and a fairly high portion of arterial roads, especially trunk highways and downtown streets.

Based on the speed map, each link's average travel time can be computed. Then estimated travel times of a route is the sum of average travel times of all links along that route. Consequently, the shortest paths via I-35W bridge is identified for stayers, named "new-after-route". Provided a new-after-route, the travel time saving for each stayer can be estimated from the speed map.

Remark. Subjects' commuting times are not the same, to make sure the following comparison is performed under the same benchmark, travel time saving proportion instead of absolute travel time saving will be used. Denote $\triangle^{(n)}$ as the travel time saving proportion by taking the new bridge for commuter n. It is computed as $\triangle^{(n)} = \frac{C_a^{(n)} - C_b^{(n)}}{C_b^{(n)}}$, where $C_b^{(n)}, C_a^{(n)}$ are estimated travel times experienced by commuter n before and after the reopening of the new bridge.

Table (3) summarizes the statistics of estimated travel time saving by using the new bridge among switchers and stayers:

Table 3: Estimated time saving statistics

Statistics		Switcher	Stayer	Total
Distribution	Counts	47	31	78
	Percentage (%)	60.3	39.7	100.0
Average Travel Time (minute)	Before	16.4	19.2	17.5
	After	14.5	18.2	16.0
	Difference	1.9	1.0	1.5
Average Travel Time Saving Percentage (%)	Average	13.0	5.4	10.0
	Minimum	2.6	0.4	0.4
	Maximum	34.4	25.2	34.4
	Median	10.5	3.5	7.9

5. Route switching analysis

5.1. Old-users and non-users

Subjects have different time saving by taking the new bridge which varies from 2.6% to 34.4% for switchers and from 0.4% to 25.2% for stayers. This wide range of time saving overlap between switchers and stayers results partially from drivers' heterogeneity.

Among 78 subjects, 44 used the old I-35W bridge regularly before it collapsed and 34 were not the regular old bridge users. Old bridge users are pre-disposed to use the new bridge while non-users may not use it even it could save substantial travel time. Therefore, old-users and non-users should display different route choice behavior in response to the addition of the new bridge.

In the following, we will further divide switchers and stayers based on whether they are "old-users" or "non-users". Denote y as the indicator of stayer (i.e., $y = 0$) or switcher (i.e., $y = 1$) and U as the indicator of non-user (i.e., $U = 0$) or old-user (i.e., $U = 1$). The frequency of stayers and switchers for non-users and old-users are summarized in Table (4).

Table 4: Contingency table of subjects' categories

	Non-user (U=0)	Old-user (U=1)	Total
Stayer (y=0)	23	8	31
Switcher (y=1)	11	36	47
Total	34	44	78

Figure (4) illustrates the boxplot of estimated time saving proportion statistics for two groups (non-user and old-user). The dots are data which are outside third quartile and represent outliers. Overall the mean and the median travel time savings for switchers are higher than those of stayers. The mean travel time savings for non-users are slightly higher than those for old-users.

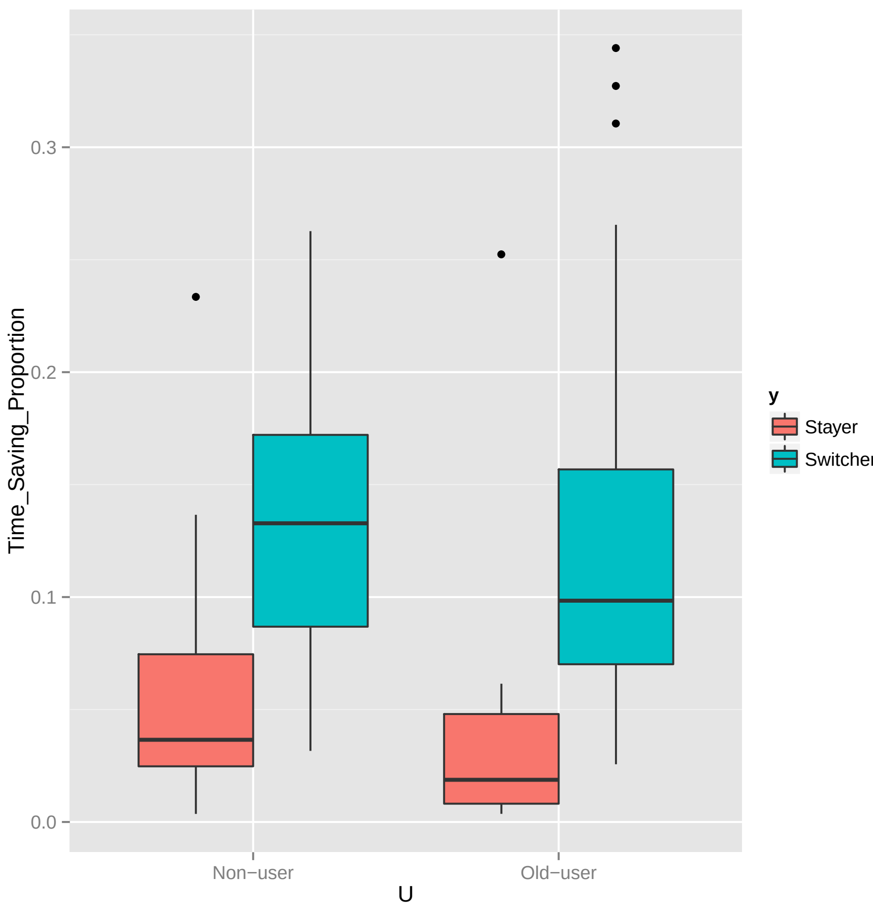


Figure 4: Boxplot of travel time saving proportions

5.2. Factors contributing to route switching

The path set for traveller $n, n = 1, \cdots, 78$ on day t before addition of the I-35W bridge is $\mathcal{P}_{nt}$. The chosen route for traveller n at time t is denoted as $A_{nt} = r$. Assume at time $t + 1$, the I-35W bridge was rebuilt. The new route due to addition of the I-35W bridge is r'_{nt}. Accordingly, the new path set enlarged by addition of the I-35W bridge is $\tilde{\mathcal{P}}_{nt} = \{\mathcal{P}, r'_{nt}\}$. Therefore, the probability of switching to the new route and the probability of staying on the current route are computed respectively as follows:

$$P(y^{(n)} = 1) = P(A_{n,t+1} = r'_{nt} | A_{nt} = r_{nt}, U_n), \tag{5.1a}$$

$$P(y^{(n)} = 0) = P(A_{n,t+1} = r_{nt} | A_{nt} = r_{nt}, U_n). \tag{5.1b}$$

where $U_n = 1$ represents that commuter n is an old-user.

$P(A_{n,t+1} = r'_{nt} | A_{nt} = r_{nt}, U_n)$ depends on the time saving by taking the new route r'_{nt} and whether commuter n has used this bridge before. Assume the log ratio of switching over staying is a linear function of time saving and commuter's group, thus a logit model is formulated:

$$\log \frac{P(y^{(n)} = 1)}{P(y^{(n)} = 0)} = \beta_0 + \beta_1 * \log(\triangle^{(n)}) + \beta_2 U_n, \tag{5.2}$$

where $\beta_0, \beta_1, \beta_2$ are regression coefficients and need to be estimated from the data. We use logarithm of time saving proportions here because time saving percentage varies between 0% and 100% and rescaling will facilitate parameter estimation.

Given 78 subjects' choices of switching or staying along with their characteristics, we have $\{y_1, \cdots, y_{78}\}$ and the predictors are $\log(\triangle^{(n)}), n = 1, \cdots, 78$ and $U_n, n = 1, \cdots, 78$. The likelihood function is: $L = \prod_{n=1}^{78} P(y^{(n)} = 1)P(y^{(n)} = 0)$. The maximum likelihood method (i.e., logit regression) is conducted to estimate parameters β_0, β_1 and results are as follows:

	Estimate	Std. Error	t-value	Pr(> |t|)
(Intercept)	4.15	1.27	3.27	0.001 **
log(△)	1.85	0.48	3.828	0.000 ***
U	2.73	0.73	3.74	0.000 ***

** Statistically significant at .5% level

*** Statistically significant at .1% level

All three parameters are significantly different from zero at 0.5% significance level. The goodness-of-fit measure by using Chi Square test gives p-value of $1.9e - 5$.

$\beta_1 = 1.85$ indicates that the log ratio of switching versus staying, i.e., $\log\left(\frac{P(y=1)}{P(y=0)}\right)$, increases by 1.85 if there is one unit increase in the logarithm of time saving brought by a new route. $\beta_2 = 2.73$ indicates that the log ratio of switching versus staying for an old-user is 2.73 times higher than that for a non-user.

Given a certain time saving, old-users have higher probability of switching than non-users. Therefore these two groups display different route switching characteristics in response to the addition of a new link.

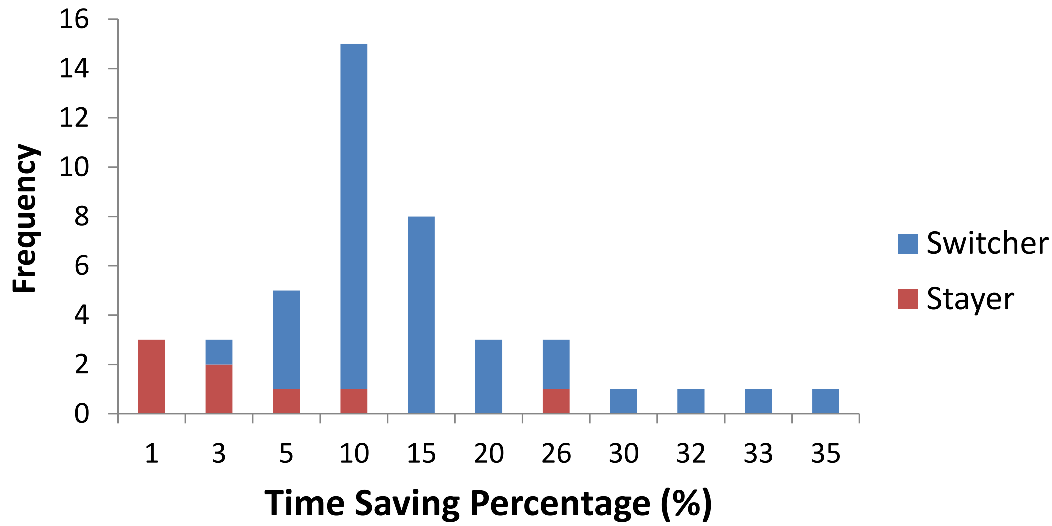
6. Indifference band estimation

When the network topology remains the same for a long enough period of time, the traffic flow pattern stabilizes and therefore travelers' route choice decisions are usually stable (Zhu and Levinson 2012), implying that they do not switch. Major network disruptions force travelers to search for new routes. Network restorations allow travelers to stay on the old route or switch to new routes, without any requirement they change.

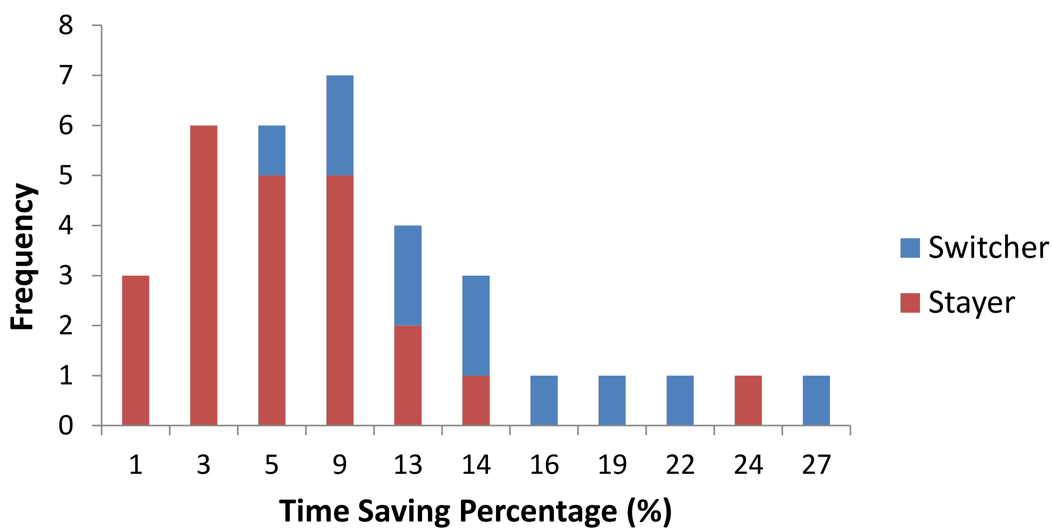
Several assumptions are made regarding boundedly rational route switching in this analysis:

- The network stabilized at an equilibrium before the new bridge was rebuilt;
- A commuter will decide to switch to the new bridge based on the principle of bounded rationality.
- A new equilibrium is reached at the end of our GPS study period.

For old-users and non-users, subjects have different time saving ranges by taking the new bridge. We divide this time saving range into $I = 11$ bins ($i = 1, \cdots, 11$) respectively. The total number of switchers and stayers for each bin can be calculated. Figure (5) illustrates the distribution of switchers and stayers for each bin for old-users. Figure (5a) show the frequencies within each bin. As the time saving increases, generally speaking, the percentage of stayers gradually decreases to zero (with only one outlier). Similar analysis can be applied to non-users shown in Figure (5b).



(a) Old-users



(b) Non-users

Figure 5: Travel time saving distribution for both switchers and stayers among old-users and non-users respectively

If travel time were the only factor which impacting route choice, classical perfect rationality cannot explain this phenomenon. Because if this were the case, everybody should immediately switch as long as the time saving is greater than zero. This may be caused by stickiness of the driving habit (Zhu and Levinson

2012). Therefore we propose that travelers are boundedly rational in route switching. Accordingly a travel time saving threshold, termed 'indifference band', is defined to capture this driving inertia. The boundedly rational user equilibrium is reached when no traveler can reduce his travel time by an indifference band by unilaterally changing routes.

6.1. Unsupervised learning

According to Figure (5), when time saving is higher than 10%, everybody tends to use the new bridge (regardless of the outlier). Thus an estimate of the indifference band for old-users is 10%. Accordingly, when the new bridge can save at least 10% travel time, 17 out of 18 subjects (i.e., the number of old-users whose time saving is greater than 10%) switched, which can capture behavioral change of 94% subjects. Similarly, an estimate of the indifference band for non-users is 14%, meaning when the new bridge saves at least 14% travel time, 4 out of 5 non-users switched and the estimation accuracy is 80.0%.

This method belongs to unsupervised learning and its disadvantage is that outlier is not considered.

6.2. Logit regression model formulation

In this section, we assume that the indifference band is a deterministic constant for old-users and non-users respectively. However, commuters may not perceive travel time accurately.

As indicated in the stimulus-response model, several biological experiments (Clark 1933; Hemmingsen 1933) verified that no response occurs unless the logarithm of stimulus exceeds some threshold. In our context, the new bridge serves as a stimulus and travelers decide to choose it or not in response. Therefore, the logarithm of time saving, denoted as $\log(\triangle^{(n)})$ will be adopted.

Travel time savings is estimated from GPS data and speed map. However, drivers may perceive it with some error (Parthasarathi et al. 2013). Denote $\hat{\triangle}^{(n)}$ as the logarithm of commuter n's perceived travel time saving, which is random:

$$\hat{\triangle}^{(n)} = \beta \log(\triangle^{(n)}) + \eta, \tag{6.1}$$

where η is a standard normal random variable, i.e., $\eta \sim N(0,1)$ with cumulative distribution function $\Phi_\eta(x)$.

Commuters will not switch routes unless the logarithm of the perceived travel time saving is greater than the logarithm of the indifference band, i.e.,

$$y^{(n)} = \begin{cases} 1, \text{ if } \hat{\triangle}^{(n)} > \log(\varepsilon^*); \\ 0, \text{ if } \hat{\triangle}^{(n)} \leqslant \log(\varepsilon^*). \end{cases} \tag{6.2}$$

where,
$y^{(n)}$: a binary indicator for commuter n. It equals one if commuter n switches to the new bridge and zero otherwise;
ε^*: the indifference band.

Therefore the probability of switching for commuter n is then computed as:

$$\begin{aligned} P(y^{(n)} = 1 | \triangle_n, U_n) &= P(\hat{\triangle}^{(n)} > \log(\varepsilon^*) | \triangle_n, U_n) = P(\beta \log(\triangle^{(n)}) + \eta > \log(\varepsilon^*) | \triangle_n, U_n) \\ &= P(\eta > \log(\varepsilon^*) - \beta \log(\triangle^{(n)}) | \triangle_n, U_n) \\ &= 1 - \Phi_\eta(\beta_0 + \beta_1 \log(\triangle^{(n)})) \end{aligned} \tag{6.3}$$

where $\beta_0 = \log(\varepsilon^*)$, $\beta_1 = -\beta$.

Using probit regression, coefficients are estimated separate for old-users and new-users (Table 5):

Table 5: Probit regression coefficients

	Estimate	Std. Error	t-value	Pr(> $\|t\|$)
Old-users				
(Intercept)	-2.42	1.00	-2.42	0.016 *
log(△)	-1.08	0.38	-2.81	0.005 **
Non-users				
(Intercept)	-3.51	0.91	-3.84	0.000 ***
log(△)	-0.92	0.30	-3.06	0.002 **

* Statistically significant at 5% level
** Statistically significant at 1% level
*** Statistically significant at 0.1% level

To illustrate, the mean indifference band for old-users is $\varepsilon^* = \exp(-2.42) = 8.9\%$ with the 97.5% confidence interval as $[1.1\%, 53.8\%]$. $\beta_1 = -1.08$ indicates that the log ratio of switching versus staying, i.e., $\log\left(\frac{P(y=1)}{P(y=0)}\right)$, increases by 1.08 if there is one unit increase in the logarithm of time saving brought by a new route.

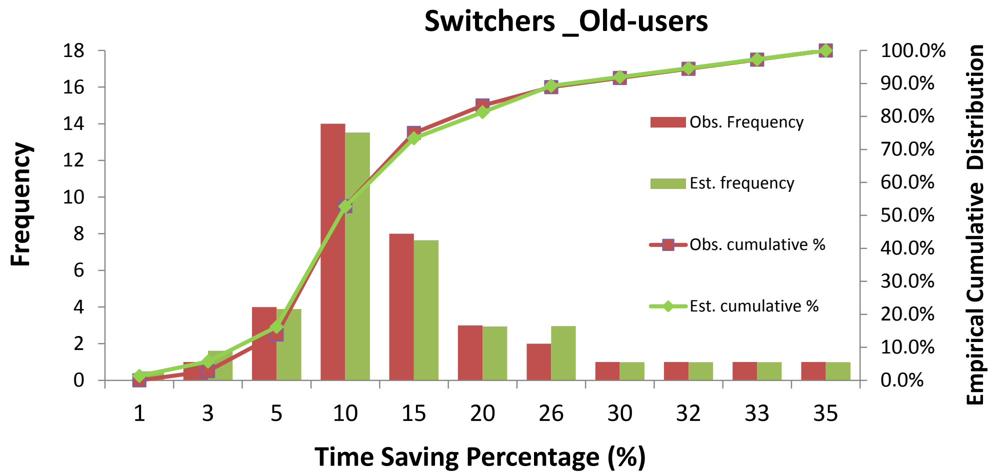
6.2.1. Estimation

To estimate the number of switchers and stayers given $\triangle$, we divide the time saving range ($0 \sim 35\%$ for old-users and $0 \sim 27\%$ for non-users respectively) into bins with 1% increment, denoted as the j^{th} bin ($j = 1, \cdots, 35$ for old-users and $j = 1, \cdots, 27$ for non-users). For each $\triangle_j$, $P(y^{(n)} = 1 | \triangle_j^{(n)}, U_n)$ can be computed according to Equation (6.3). The expected number of switchers for the j^{th} bin can be computed as:

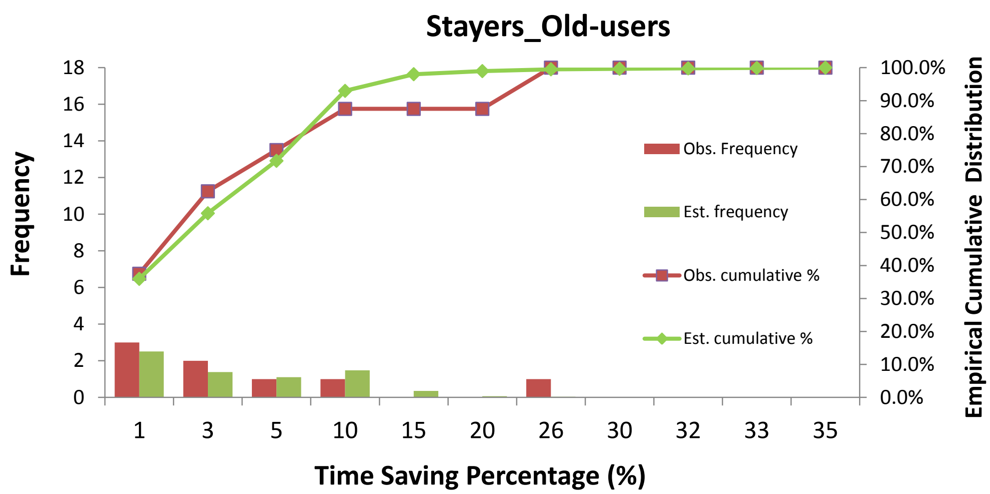
$$\hat{N}^{(j)}_{switcher} = N^{(j)} P(y = 1 | \triangle{=}\triangle_j, U) \tag{6.4}$$

where,
$N^{(j)}$: the number of observations (i.e., switchers plus stayers) for the j^{th} bin;
$\triangle_j$: the critical time saving thresholds for the j^{th} bin.

Note that we also divide the range into $I = 11$ bigger bins (each bin is denoted as the i^{th} bin) when we calculate frequencies of switchers and stayers in Figure (5). Then we can aggregate the total expected number of switchers and stayers within the j^{th} bin for each $i^{th}, i = 1, \cdots, 11$ bin. Figures (6-7) illustrate frequency and cumulative distribution of switchers and stayers from estimation and observation respectively.

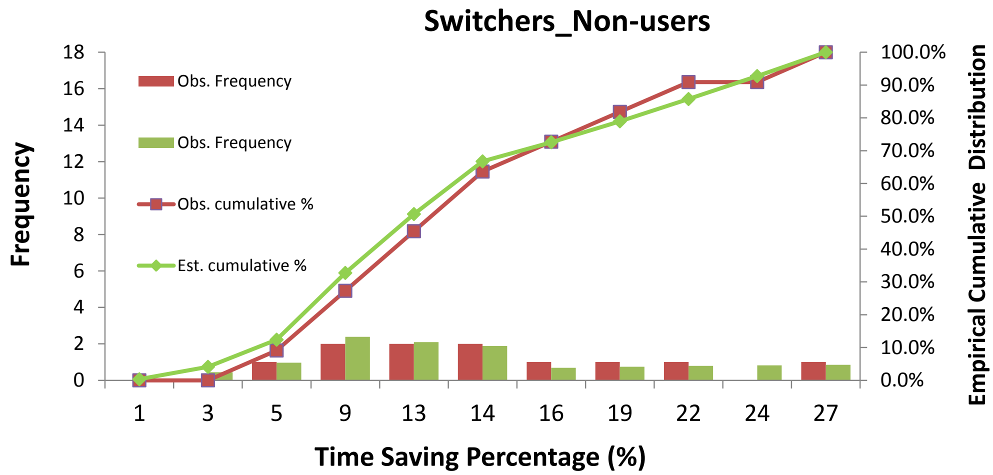


(a) Switchers among old-users

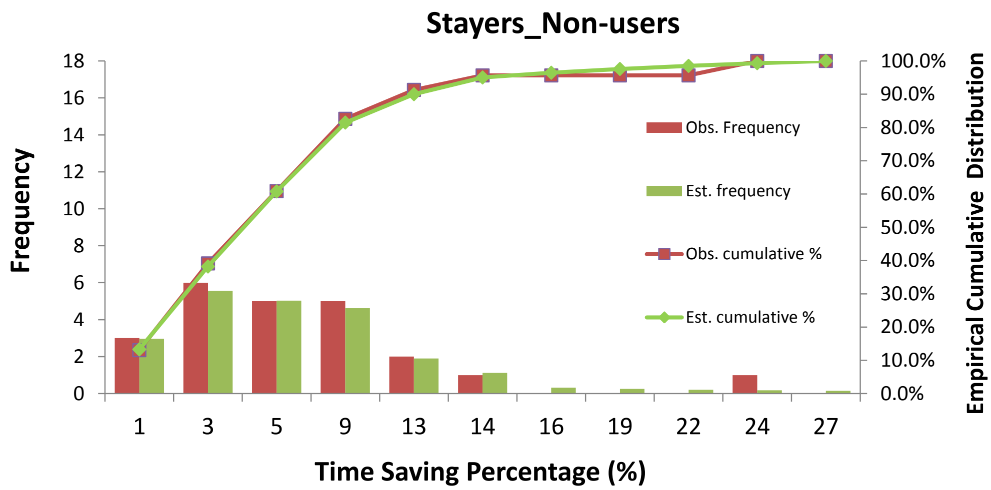


(b) Stayers among old-users

Figure 6: Frequency and cumulative distribution for old-users



(a) Switchers among non-users



(b) Stayers among non-users

Figure 7: Frequency and cumulative distribution for non-users

Define the mean square error as $MSE = \frac{1}{N}\sum_{i=1}^{I}\left(\hat{N}^{(i)}_{switcher} - N^{(i)}_{switcher}\right)^2$, where $\hat{N}^{(i)}_{switcher}$ is the estimated number of switchers for the j^{th} bin and $N^{(i)}_{switcher}$ is the observed number of switchers, N is the total number of subjects. MSE is 17.4% for old-users and 11.5% for non-users respectively. The estimated frequency and cumulative percentage of switchers match the observed ones well and indicates that the proposed model can capture the switching pattern in data.

7. Conclusions and future research directions

This paper proposes a boundedly rational route switching model to explain the observation that fewer commuters use the new I-35W bridge in Minneapolis. The boundedly rational route switching model assumes that commuters will not switch to the new bridge unless the time saving by taking the new bridge is higher than an indifference band. The route choice behavioral data collected from a GPS travel survey is used to estimate the indifference band parameter. The estimation result shows that more travelers switch as time saving increases. In addition, commuters who were regular old bridge users manifested different route choice behavior, compared to those who never had the experience of using the old bridge. Therefore the subjects are further divided into two groups: old-users and non-users. Within each group, the indifference band is estimated by using probit regression. The estimated numbers of switchers and stayers match the observed data well and old-users have a lower indifference threshold than non-users.

This study provides the insight into the route choice behavior in the Minneapolis-St. Paul region and is the first empirical study using GPS field data to estimate bounded rationality parameter. However, this study can be generalized as follows. Travelers behave heterogeneously in route choice. Travel time and using history may not be the only factors affecting route choices and other factors play important roles. Moreover, the collapse and reopening of bridges are fortunately rare events. To further study route changes in response to the change in road networks, road closures for construction may be good substitutes because they are more frequent. Route choice data should be collected from these events to extend the findings of this paper.

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